Auction No. 66 - Advanced Wireless Services (AWS-1) DA 06-238

Attachment A - Economic Area (EA) Licenses

Market	<u> </u>		1.117/	Channel		Bandwidth	Bidding	Upfront	Minimum
Sumber	Description	License Number	Frequencies (MHz)	Block	Population	(MHz)	Units	Payment	Opening Bid
	Na (LA) (en Basie) Policinio A 727 DEA			1311 K.K.		('110')	era management and	T.i.yiiiCiiC	
BEA165	Redding CA-OR	AW-BEA165-B	1720-1730 / 2120-2130	В	336,820	20	337,000	\$337,000	\$337,000
BEA165	Redding CA-OR	AW-BEA165-C	1730-1735 / 2130-2135	С	336,820	10	168,000	\$168,000	\$168,000
BEA 166	Eugene-Springfield OR-CA	AW-BEA166-B	1720-1730 / 2120-2130	В	791,776	20	792,000	\$792,000	\$792,000
BEA 166	Eugene-Springfield OR-CA	AW-BEA166-C	1730-1735 / 2130-2135	С	791,776	10	396,000	\$396,000	\$396,000
BEA 167	Portland-Salem OR-WA	AW-BEA167-B	1720-1730 / 2120-2130	В	2,883,737	20	2,884,000	\$2,884,000	\$2,884,000
BEA167	Portland-Salem OR-WA	AW-BEA167-C	1730-1735 / 2130-2135	С	2,883,737	10	1,442,000	\$1,442,000	\$1,442,000
BEA168	Pendleton OR-WA	AW-BEA168-B	1720-1730 / 2120-2130	В	200,681	20	201,000	\$201,000	\$201,000
BEA 168	Pendleton OR-WA	AW-BEA168-C	1730-1735 / 2130-2135	С	200,681	10	100,000	\$100,000	\$100,000
BEA 169	Richland-Kennewick-Pasco WA	AW-BEA169-B	1720-1730 / 2120-2130	В	677,674	20	678,000	\$678,000	\$678,000
BEA 169	Richland-Kennewick-Pasco WA	AW-BEA169-C	1730-1735 / 2130-2135	С	677,674	10	339,000	\$339,000	\$339,000
BEA170	Seattle-Tacoma-Bremerton WA	AW-BEA170-B	1720-1730 / 2120-2130	В	4,135,291	20	4,135,000	\$4,135,000	\$4,135,000
BEA 170	Seattle-Tacoma-Bremerton WA	AW-BEA170-C	1730-1735 / 2130-2135	C	4,135,291	10	2,068,000	\$2,068,000	\$2,068,000
BEA171	Anchorage AK	AW-BEA171-B	1720-1730 / 2120-2130	В	626,932	20	627,000	\$627,000	\$627,000
BEA 171	Anchorage AK	AW-BEA171-C	1730-1735 / 2130-2135	С	626,932	10	313,000	\$313,000	\$313,000
BEA172	Honolulu HI	AW-BEA172-B	1720-1730 / 2120-2130	В	1,211,537	20	1,212,000	\$1,212,000	\$1,212,000
BEA 172	Honolulu HI	AW-BEA172-C	1730-1735 / 2130-2135	C	1,211,537	10	606,000	\$606,000	\$606,000
BEA 173	Guam-Northern Mariana Islands	AW-BEA173-B	1720-1730 / 2120-2130	В	224,026	20	224,000	\$224,000	\$224,000
BEA 173	Guam-Northern Mariana Islands	AW-BEA173-C	1730-1735 / 2130-2135	С	224,026	10	112,000	\$112,000	\$112,000
BEA174	Puerto Rico-US Virgin Islands	AW-BEA174-B	1720-1730 / 2120-2130	В	3,917,222	20	3,917,000	\$3,917,000	\$3,917,000
BEA 174	Puerto Rico-US Virgin Islands	AW-BEA174-C	1730-1735 / 2130-2135	С	3,917,222	10	1,959,000	\$1,959,000	\$1,959,000
BEA175	American Samoa	AW-BEA175-B	1720-1730 / 2120-2130	В	57,291	20	57,000	\$57,000	\$57,000
BEA175	American Samoa	AW-BEA175-C	1730-1735 / 2130-2135	С	57,291	10	29,000	\$29,000	\$29,000
BEA 176	Gulf of Mexico	AW-BEA176-B	1720-1730 / 2120-2130	В	-	20	40,000	\$40,000	\$40,000
BEA 176	Gulf of Mexico	AW-BEA176-C	1730-1735 / 2130-2135	Ć	_	10	20,000	\$20,000	\$20,000

Auction Total 352 BEA Licenses 428,498,000 \$428,498,000 \$428,498,000

Auction No. 66 - Advanced Wireless Services (AWS-1) DA 06-238

Attachment A - Regional Economic Area Grouping (REAG) Licenses

Market				Channel		Bandwidth		Upfront	Minimum
Number	Description	License Number	Frequencies (MHz)	Block	Population	(MHz)	Bidding Units	Payment	Opening Bid
				Fire Williams Care		alega di menangga		Miles Mark	
REA001	Northeast	AW-REA001-D	1735-1740 / 2135-2140	D	50,058,090	10	25,029,000	\$25,029,000	\$25,029,000
REA001	Northeast	AW-REA001-E	1740-1745 / 2140-2145	Ē	50,058,090	10	25.029.000	\$25,029,000	\$25,029,000
REA001	Northeast	AW-REA001-F	1745-1755 / 2145-2155	F	50,058,090	20	50,058,000	\$50,058,000	\$50,058,000
REA002	Southeast	AW-REA002-D	1735-1740 / 2135-2140	D	49,676,946	10	24,838,000	\$24,838,000	\$24,838,000
REA002	Southeast	AW-REA002-E	1740-1745 / 2140-2145	E	49,676,946	10	24,838,000	\$24,838,000	\$24,838,000
REA002	Southeast	AW-REA002-F	1745-1755 / 2145-2155	F	49,676,946	20	49,677,000	\$49,677,000	\$49,677,000
REA003	Great Lakes	AW-REA003-D	1735-1740 / 2135-2140	D	58,178,304	10	29,089,000	\$29,089,000	\$29,089,000
REA003	Great Lakes	AW-REA003-E	1740-1745 / 2140-2145	Е	58,178,304	10	29,089,000	\$29,089,000	\$29,089,000
REA003	Great Lakes	AW-REA003-F	1745-1755 / 2145-2155	F	58,178,304	20	58,178,000	\$58,178,000	\$58,178,000
REA004	Mississippi Valley	AW-REA004-D	1735-1740 / 2135-2140	D	31,326,973	10	15,663,000	\$15,663,000	\$15,663,000
REA004	Mississippi Valley	AW-REA004-E	1740-1745 / 2140-2145	Е	31,326,973	10	15,663,000	\$15,663,000	\$15,663,000
REA004	Mississippi Valley	AW-REA004-F	1745-1755 / 2145-2155	F	31,326,973	20	31,327,000	\$31,327,000	\$31,327,000
REA005	Central	AW-REA005-D	1735-1740 / 2135-2140	D	40,343,960	10	20,172,000	\$20,172,000	\$20,172,000
REA005	Central	AW-REA005-E	1740-1745 / 2140-2145	Е	40,343,960	10	20,172,000	\$20,172,000	\$20,172,000
REA005	Central	AW-REA005-F	1745-1755 / 2145-2155	F	40,343,960	20	40,344,000	\$40,344,000	\$40,344,000
REA006	West	AW-REA006-D	1735-1740 / 2135-2140	D	49,999,164	10	25,000,000	\$25,000,000	\$25,000,000
REA006	West	AW-REA006-E	1740-1745 / 2140-2145	Е	49,999,164	10	25,000,000	\$25,000,000	\$25,000,000
REA006	West	AW-REA006-F	1745-1755 / 2145-2155	F	49,999,164	20	49,999,000	\$49,999,000	\$49,999,000
REA007	Alaska	AW-REA007-D	1735-1740 / 2135-2140	D	626,932	10	313,000	\$313,000	\$313,000
REA007	Alaska	AW-REA007-E	1740-1745 / 2140-2145	Е	626,932	10	313,000	\$313,000	\$313,000
REA007	Alaska	AW-REA007-F	1745-1755 / 2145-2155	F	626,932	20	627,000	\$627,000	\$627,000
REA008	Hawaii	AW-REA008-D	1735-1740 / 2135-2140	D	1,211,537	10	606,000	\$606,000	\$606,000
REA008	Hawaii	AW-REA008-E	1740-1745 / 2140-2145	E	1,211,537	10	606,000	\$606,000	\$606,000
REA008	Hawaii	AW-REA008-F	1745-1755 / 2145-2155	F	1,211,537	20	1,212,000	\$1,212,000	\$1,212,000
REA009	Guam, Northern Mariana Islands	AW-REA009-D	1735-1740 / 2135-2140	D	224,026	10	112,000	\$112,000	\$112,000
REA009	Guam, Northern Mariana Islands	AW-REA009-E	1740-1745 / 2140-2145	E	224,026	10	112,000	\$112,000	\$112,000
REA009	Guam, Northern Mariana Islands	AW-REA009-F	1745-1755 / 2145-2155	F	224,026	20	224,000	\$224,000	\$224,000
REA010	Puerto Rico, US Virgin Islands	AW-REA010-D	1735-1740 / 2135-2140	D	3,917,222	10	1,959,000	\$1,959,000	\$1,959,000
REA010	Puerto Rico, US Virgin Islands	AW-REA010-E	1740-1745 / 2140-2145	Е	3,917,222	10	1,959,000	\$1,959,000	\$1,959,000
REA010	Puerto Rico, US Virgin Islands	AW-REA010-F	1745-1755 / 2145-2155	F	3,917,222	20	3,917,000	\$3,917,000	\$3,917,000
REA011	American Samoa	AW-REA011-D	1735-1740 / 2135-2140	D	57,291	10	29,000	\$29,000	\$29,000
REA011	American Samoa	AW-REA011-E	1740-1745 / 2140-2145	Е	57,291	10	29,000	\$29,000	\$29,000
REA011	American Samoa	AW-REA011-F	1745-1755 / 2145-2155	F	57,291	20	57,000	\$57,000	\$57,000
REA012	Gulf of Mexico	AW-REA012-D	1735-1740 / 2135-2140	D		10	20,000	\$20,000	\$20,000
REA012	Gulf of Mexico	AW-REA012-E	1740-1745 / 2140-2145	Ė		10	20,000	\$20,000	\$20,000
REA012	Gulf of Mexico	AW-REA012-F	1745-1755 / 2145-2155	F	-	20	40,000	\$40,000	\$40,000

Auction Total 36 REAG Licenses 571,320,000 \$571,320,000 \$571,320,000

Auction No. 66 – Advanced Wireless Services (AWS-1) Attachment B

Using the Smoothed Anchoring Method to Obtain Current Price Estimates

This appendix describes the method by which bid information on packages and licenses is used to approximate a "price" associated with each license at the close of every round. These "current price estimates" ("CPEs") are then used in the next round when calculating minimum acceptable bid amounts. Specifically, for a license, this value is the CPE of the license plus a percentage. For a package, the minimum acceptable bid amount is the sum of the minimum acceptable bid amounts of its component licenses.

The current price estimates of the licenses are based on the concept that every linear optimization problem has a dual problem that provides pricing information. We begin by discussing a simplified representation of the FCC winner determination problem and then discuss its linear programming relaxation before explaining the dual problem of interest. The winner determination problem is shown in (P1):

$$\max \sum_{j \in B^t} b_j x_j$$
(P1): $s.t. \sum_{j \in B^t} a_{ij} x_j = 1$, for all $i \in L$ (1)
$$x_j \in \{0,1\}, \quad \text{for all } j \in B^t$$

where B^t is the set of considered bids in round t,

 b_j is the bid amount of bid j,

L is the set of licenses being auctioned,

$$a_{ij} = \begin{cases} 1 & \text{if license } i \text{ is in bid } j \\ 0 & \text{otherwise} \end{cases} \text{ and,}$$

$$x_{j} = \begin{cases} 1 & \text{if bid } j \text{ is in the winning set} \\ 0 & \text{otherwise} \end{cases}$$

In this formulation, x_j is an indicator variable that equals one if bid j is in the provisionally winning set and zero otherwise. Thus, the sum of the bid amounts of all provisionally winning bids produces the maximum obtainable revenue for round t. Constraints (1) ensure that each license is awarded exactly once. The constraints that ensure that a bidder's bids between rounds are mutually exclusive are not represented in (P1) since they will be ignored in the linear representation of the problem.¹

¹ These constraints will be ignored in the linear program representation since they are rarely binding in the relaxation of the integer-programming problem and because adding such constraints to the dual problem creates "degeneracy" in the solution thereby causing multiple alternative solutions.

The linear program of (P1) relaxes the restriction on the variables x_j , for all $j \in B^t$, allowing these variables to take on any value between zero and one. The linear programming representation of (P1) is shown in (P2):

$$\max \quad \sum_{j \in B'} b_j x_j$$
 (P2):
$$s.t. \quad \sum_{j \in B'} a_{ij} x_j = 1, \qquad \text{for all } i \in L$$

$$x_j \ge 0, \qquad \qquad \text{for all } j \in B'$$

The dual formulation of (P2) can be used to identify a price, π_i , for each license i, and is shown in the following linear program (P3):

(P3):
$$\min \sum_{i \in L} \pi_{i}$$

$$s.t. \sum_{i \in L} a_{ji} \pi_{i} \geq b_{j}, \text{ for all } j \in B' \setminus F$$

$$\pi_{i} \geq b_{j}, \text{ for all } j \in F$$

and i is the license index associated with bid j (3)

where $F \subset B^t$ is the set of FCC bids on each license² and, $a_{ji} = \begin{cases} 1, & \text{if bid } j \text{ contains license } i \\ 0, & \text{otherwise} \end{cases}.$

The optimal value of each variable, π_i , in (P3) corresponds to a dual price³ – often called a "shadow price" – for each constraint, i.e., each license, in (P2). The dual price of each license measures the monetary cost of not awarding the license to whom it has been provisionally assigned under the solution to (P2). Thus, this monetary cost has a clear and natural use in estimating the current price of a license given the bids considered in the current round.

Constraints (2) in (P3) ensure that the dual price of a license must be at least as large as the greatest bid made on that license. For a package, these constraints ensure that the sum of the dual prices of the licenses that make up a particular package must be at least as large as the greatest bid made on that package. Constraints (3) in (P3) ensure that if a license has not been bid on, the dual price of that license is at least as large as the FCC bid amount.

Ideally, the solution to (P2) is identical to the solution of (P1). When this occurs, the sum of the dual prices of the licenses comprising any provisionally winning bid equals the winning bid amount. However, (P2) is only an approximation to the integer problem⁴ and often overestimates

² The system maintains an FCC bid amount at some small amount less than the minimum opening bid for that license, in order to avoid ties with bids at the minimum opening bid amount.

³ We note that for non-linear problems, these dual prices are also known as Lagrange multipliers.

⁴ When the problem is a convex optimization problem, the primal and dual problems yield the same objective function values. This is called strong-duality. These conditions do not hold for integer programming problems, often resulting in a gap between the linear programming and integer programming solution values.

the maximum revenue of (P1). When this occurs, the sum of the dual prices of the licenses in at least one provisionally winning bid will be greater than the respective bid amount. Thus, using the dual prices of (P3) can result in minimum acceptable bid amounts that are too high.

We propose to resolve this issue by using pseudo-dual prices,⁵ rather than the dual prices of (P3). These pseudo-dual prices are obtained by forcing the sum of the dual prices of the licenses comprising a provisionally winning bid to equal its respective bid amount. For example, suppose there are two bids in the provisionally winning set in round t: a bid on license A for \$10 and a bid on package BC for \$25. The pseudo-dual price of A would exactly equal \$10 and the sum of the pseudo-dual prices of B and C would exactly equal \$25. These restrictions ensure that the sum of the pseudo-dual prices equals the maximum revenue for the round (e.g. \$35) and that minimum acceptable bid amounts reflect the bid amounts of bids in the provisionally winning set.

Pseudo-dual prices for each license i, denoted π_i , satisfy the following constraints:

$$\sum_{i \in L} a_{ji} \pi_i + \delta_j \ge b_j, \text{ for all } j \in B^t \setminus (W^t \cup F)$$
(4)

$$\sum_{i \in L} a_{ji} \pi_i = b_j, \quad \text{for all } j \in W^t$$
 (5)

$$\pi_i \geq b_j$$
, for all $j \in F \setminus (W^t \cap F)$

and i is the license index associated with bid j (6)

$$\delta_j \ge 0,$$
 for all $j \in B^t \setminus (W^t \cup F)$ (7)

where $W \subset B^i$ is the provisionally winning bid set in round t and, δ_j is a slack variable that represents the difference between the bid amounts of non-winning bid j and the sum of pseudo-dual prices of the licenses contained in non-winning bid j

Constraints (5) ensure that for each provisionally winning bid, the sum of the dual prices of the licenses comprising that bid equal its respective bid amount. This new restriction requires that we ease restriction (2) in (P3) for non-winning bids in order to ensure that a feasible solution exists. Constraints (4) provide this needed slack. Constraints (6) are equivalent to constraints (3) in (P3) and constraints (7) force the slack variables to be non-negative.

Satisfying constraints (5) implies that the sum of the pseudo-dual prices always yields the maximum revenue for the round. There are likely to be many sets of pseudo-dual prices that satisfy this constraint set. For instance, in the example provided earlier, the pseudo-dual prices of B and C might be any two numbers that together sum to \$25.

By keeping constraints (4)-(7), we have the flexibility to choose an objective function that will help in selecting among multiple solutions while still ensuring that the sum of the pseudo-dual prices yields the maximum revenue of the round. We would like an objective function that minimizes the values of the slack variables δ_j , for all $j \in B^t \setminus (W^t \cup F)$ in order to obtain pseudo-

⁵ In our research we found this term first applied to auction pricing in the paper by Rassenti, Smith and Bulfin (1982), "A combinatorial auction mechanism for airport slot allocation," *Bell Journal of Economics*, vol. 13, pp. 402-417.

dual prices that are close to the dual prices of (P3). We have tested a number of alternative objective functions:

- 1. Minimization of the maximum δ_j for all $j \in B^t \setminus (W^t \cup F)$ followed by maximization of the minimum π_i for all i in license set L, in an iterative manner. (DeMartini, Kwasnica, Ledyard and Porter, 1999)
- 2. Minimization of the sum of the squares of δ_j for all $j \in B^t \setminus (W^t \cup F)$. (also DeMartini, Kwasnica, Ledyard and Porter, 1999)
- 3. Minimization of the sum of the δ_j for all $j \in B' \setminus (W' \cup F)$ using a "centering" algorithm⁶ to solve, essentially finding an average among all sets of optimal pseudodual prices.

In testing the above alternatives, we frequently observed instances where the pseudo-dual price of a license significantly changed from round to round. We acknowledge that prices of licenses should be allowed to reflect real changes, both increases and decreases, in the way bidders value the licenses over time. However, we believe that large oscillations in minimum acceptable bid amounts for the same bid that are due to irrelevant factors such as multiple optimal solutions, can be confusing to bidders. We have therefore chosen a method that attempts to balance minimizing the slack variables and reducing the fluctuations in pseudo-dual prices from round to round. This method requires solving two optimization problems, the first of which is alternative 3 above, which we present as (P4):

$$\Omega^{*} = \min \sum_{j \in B' \setminus (W^{t} \cup F)} \delta_{j}$$

$$st. \sum_{i \in L} a_{ji} \pi_{i} + \delta_{j} \geq b_{j}, \text{ for all } j \in B^{t} \setminus (W^{t} \cup F)$$

$$\sum_{i \in L} a_{ji} \pi_{i} = b_{j}, \quad \text{ for all } j \in W^{t}$$

$$\pi_{i} \geq b_{j}, \quad \text{ for all } j \in F \setminus (W^{t} \cap F)$$

$$\text{ and } i \text{ is the license index associated with bid } j$$

$$\delta_{j} \geq 0, \quad \text{ for all } j \in B^{t} \setminus (W^{t} \cup F)$$

Since multiple optimal solutions can exist to (P4) we solve a second optimization problem that chooses a solution in a way that reduces the magnitude of price fluctuations between rounds. Specifically, we use an objective function that applies the concepts of exponential smoothing⁷ to choose among alternative pseudo-dual prices with the additional constraint on the problem that the sum of the slack variables equals Ω^* (the optimal value of (P4)). This objective function minimizes the sum of the squared deviations of the resulting pseudo-dual prices in round t, from their respective smoothed prices in round t-1.8 At the start of the auction, we use the minimum opening bid prices as the prior smoothed prices. Since these opening prices are based on

⁶ The centering algorithm used in this testing was the barrier method available in CPLEX, a commercial optimization package.

⁷ Exponential amost in a few interests in the commercial optimization package.

⁷ Exponential smoothing often is used in determining minimum acceptable bids in FCC auctions. See Attachment C of this Public Notice.

⁸ This objective function is a convex, quadratic function. This quadratic optimization problem is solved using the quadratic simplex method.

bandwidth and population, the pricing algorithm begins with a priori information about the differences among licenses.

Let π_i^t be the pseudo-dual price of license i in round t. The smoothed price for license i in round t is calculated using the following exponential smoothing formula:

$$p_i^t = \alpha \pi_i^t + (1 - \alpha) p_i^{t-1}$$

where p_i^{t-1} is the smoothed price in round t-1, $0 \le \alpha \le 1$, and p_i^0 = the minimum opening bid amount for license i.

Consistent with prior practice of the Commission, a weighting factor of $\alpha = 0.5$ has been chosen but can change, as the Commission requires.

The following quadratic program (QP) will find the pseudo-dual price, π_i^t , for each license i in round t that minimizes the sum of the squared deviations from the respective smoothed prices in round t-1 while ensuring that the pseudo-dual prices sum up to the provisionally winning bid amounts and that the sum of the slack variables is minimized.

$$\begin{aligned} &\min & \sum_{i \in L} (\pi_i^t - p_i^{t-1})^2 \\ &s.t & \sum_{i \in L} a_{ji} \pi_i^t + \delta_j \geq b_j, \text{ for all } j \in B^t \setminus (W^t \cup F) \\ & \sum_{i \in L} a_{ji} \pi_i^t = b_j, &\text{ for all } j \in W^t \\ & \sum_{j \in B' \setminus (W' \cup F)} \delta_j = \Omega^* \\ & \pi_i^t & \geq b_j, &\text{ for all } j \in F \setminus (W^t \cap F) \\ & &\text{ and } i \text{ is the license index associated with bid } j \\ & \delta_j & \geq 0 \text{ , } &\text{ for all } j \in B^t \setminus (W^t \cup F) \end{aligned}$$

where p_i^{t-1} is known and treated as a constant within the optimization.

Among alternative prices that satisfy all constraints, the objective function of this optimization problem chooses one that forces the pseudo-dual prices to be as close as possible to the previous round's smoothed price. Thus, we call this the *Smoothed Anchoring Method* since we "anchor" on the smoothed prices when solving for the pseudo-dual prices. We define the CPE for license i in round t as the pseudo-dual price, π_i^t , obtained by solving (QP).

⁹ Once the pseudo-dual prices, π_i^t , have been determined, the smoothed prices, p_i^t , can be calculated and used for solving (QP) in round t+1.

The minimum acceptable bid amount for a license in round t+1 will be the CPE of the license, as calculated above, plus a percentage. For a package, the minimum acceptable bid amount will be the sum of the minimum acceptable bid amounts of the licenses that make up the package.

Auction No. 66 – Advanced Wireless Services (AWS-1) Attachment C

Smoothing Formula Equations

$$A_i = (C * B_i) + ((1-C) * A_{i-1})$$

$$I_{i+1}$$
 = smaller of ((1 + A_i) * N) and M

$$\mathbf{X}_{i+1} = \mathbf{I}_{i+1} * \mathbf{Y}_i$$

where.

 A_i = activity index for the current round (round i)

C = activity weight factor

B_i = number of bidders submitting bids on the licenses in the current round (round i)

 A_{i-1} = activity index from previous round (round i-1), A_0 is 0

 I_{i+1} = percentage increment for the next round (round i+1)

N = minimum percentage increment or percentage increment floor

M = maximum percentage increment or percentage increment ceiling

 X_{i+1} = dollar amount associated with the percentage increment

Y_i = provisionally winning bid amount from the current round

Examples

License 1

$$C=0.5$$
, $N=0.1$, $M=0.2$

Round 1 (2 bidders submitting bids, provisionally winning bid = \$1,000,000)

1. Calculation of percentage increment for round 2 using the smoothing formula:

$$A_1 = (0.5 * 2) + (0.5 * 0) = 1$$

 $I_2 =$ The smaller of $((1 + 1) * 0.1) = 0.2$ or 0.2 (the maximum percentage increment)

2. Calculation of dollar amount associated with the percentage increment for round 2 (using I₂ from above):

$$X_2 = 0.2 * $1,000,000 = $200,000$$

3. Minimum acceptable bid amount for round 2 = 1,200,000

Round 2 (3 bidders submitting bids, provisionally winning bid = \$2,000,000)

1. Calculation of percentage increment for round 3 using the smoothing formula:

$$A_2 = (0.5 * 3) + (0.5 * 1) = 2$$

 $I_3 =$ The smaller of $((1 + 2) * 0.1) = 0.3$ or 0.2 (the maximum percentage increment)

2. Calculation of dollar amount associated with the percentage increment for round 3 (using I₃ from above):

$$X_3 = 0.2 * $2,000,000 = $400,000$$

3. Minimum acceptable bid amount for round 3 = \$2,400,000

Round 3 (1 bidder submitting bids, provisionally winning bid = \$2,400,000)

1. Calculation of percentage increment for round 4 using the smoothing formula:

$$A_3 = (0.5 * 1) + (0.5 * 2) = 1.5$$

 $I_4 =$ The smaller of $((1 + 1.5) * 0.1) = 0.25$ or 0.2 (the maximum percentage increment)

2. Calculation of dollar amount associated with the percentage increment for round 4 (using I₄ from above):

$$X_4 = 0.2 * \$2,400,000 = \$480,000$$

3. Minimum acceptable bid amount for round 4 = \$2,880,000